GOVERNMENT OF INDIA
CENTRAL WATER COMMISSION
CENTRAL TRAINING UNIT

HYDROLOGY PROJECT

TRAINING OF TRAINERS
IN
HYDROMETRY

UNDERSTANDING STAGE -DISCHARGE RELATIONS

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PUNE - 411 024

TABLE OF CONTENTS

1. MODULE CONTEXT

This module is a part of the 'Training in Hydrometry' for middle level engineers. This module is one of the two modules on 'Stage-Discharge Relations'. The two modules are :

Module	Code	Subject Contents
1. Understanding Stage - Discharge Relation		– Introduction to Stage - Discharge ratings, and Correlation and Regression – Classification of controls – Characteristics and Extrapolation of rating curves – Shifts in discharge ratings
2. How to analyse Stability of SD relation		– Fitting of curve for S-D relations – Testing the significance of curve fitting – Drawing of confidence limits – IS Code procedures

2. MODULE INFORMATION

Title	:	Understanding Stage-Discharge Relation
Target Group	:	Middle Level Engineers
Duration	:	90 minutes
Objectives	:	After training, the officers would be able to understand the concept of Stage-Discharge Relation and impart training to Supervisors and Junior Staff
Key Concepts	:	– Correlation and Regression – Method of least squares – Classification of controls – Rating Curve Extrapolation – Shifts in ratings
Training methods	:	Lecture, discussions & questioning
Training aids	:	Overhead Projector, Transperancies, blackboard, Examples of Regression Analysis
Handout	:	Main text and Example

3. SESSION PLAN

Activity	Time
1. Introduction to Stage - Discharge Relations	5 minutes
2. Discuss about correlation & Regression analysis	5 minutes
3. Talk about classification of controls	10 minutes
4. Brief discussion	5 minutes
5. Briefly explain about characteristics of Rating Curve	10 minutes
6. Discuss the example given in handout	10 minutes
7. Details about methods of extrapolation	20 minutes
8. Explain about shits in discharge ratings	10 minutes
9. Questions & discussions for testing	15 minutes
	90 minutes

INTRUCTORS NOTE

UNDERSTANDING STAGE-DISCHARGE RELATIONS

1.0 INTRODUCTION

'Discharge Rating' is usually defined as the relation between the river stage and discharge. The terms 'rating', 'rating curve', 'station rating', and 'stage-discharge relation ' are synonymous with the term 'discharge rating'.

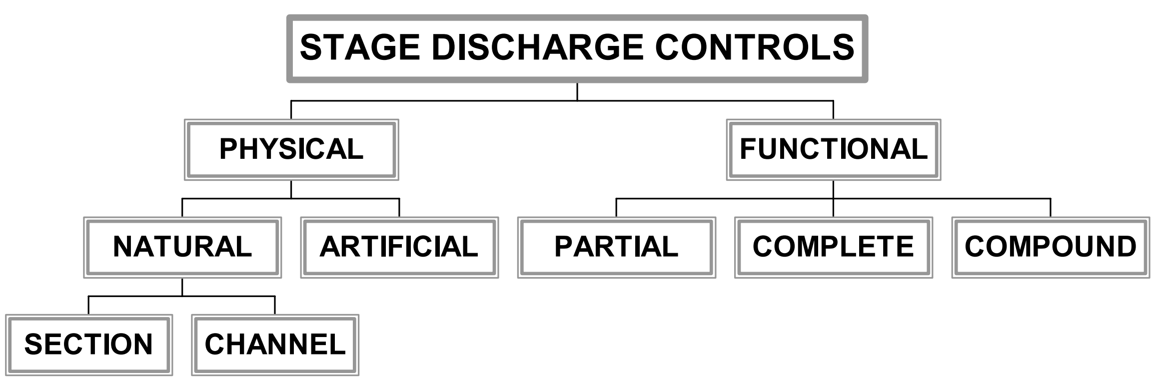
The measured discharge is plotted against the corresponding stage to define the rating curve; the plot could be in rectangular coordinates or logarithmic. The discharge is plotted as the abscissa and the stage as the ordinate in rectangular coordinates.

While it is much easier to observe the stages of a river, the discharge observation by actually measuring the depth and velocity of the flow is cumbersome and expensive. Hence the data is often collected in the form of stages and is converted into discharge with the help of an established stage-discharge relation as the data in the form of discharge may be more meaningful and desired by the users in their analysis. A good and stable stage- discharge relation is very much essential for estimating the discharges accurately from the observed stages. Though the World Meteorological Organisation (WMO) recommends a minimum of ten discharge measurements per year, it may be necessary to have regular daily observations of stage and discharge simultaneously for an initial period in order to establish a reliable stage- discharge relation at a hydrological observation station.

2.0 STAGE-DISCHARGE CONTROLS

The stage-discharge relation at a gauging station is often controlled by the section or reach of the channel down stream of the station gauge line or certain other physical element. A stage-discharge control could be defined as the physical element or a combination of elements that control the relation between the stage and discharge at the section of measurement of the flow. This element could be in the form of a rock outcrop or a natural riffle, or any other physical feature due to the presence of which the water level on the upstream side of the section increases and a good stage-discharge relation is obtained.

2.1 Classification of controls



Controls can be classified such as section control, channel control, partial control, complete control and compound control. These controls could either be natural or artificial.

2.1.1 Section Controls

The section control, as the name indicates is due to the shape of the cross-section. When the geometry of the cross -section allows the channel to be constricted which could either be from a local rise in the stream bed or a natural riffle or a rockledge outcrop, the section control plays a predominant role in stabilising the stage-discharge relation. The constriction could also be artificial as in the case of manmade encroachment (resulting in the reduction of the width of the stream), or a constructed weir, or bridge (which reduces the waterway). Section control also exists when there is a sudden break in the bed slope. Examples of such control are the head of a cascade or the brink of a waterfall etc. The section control is usually effective at low discharges and the channel controls take over at medium and high stages.

2.2 Channel Controls

A channel control is said to exist when the geometry and the roughness of a long reach of the stream, downstream of the station gauge line, controls the Stage-discharge relationship. This control usually consists of all the physical features of the channel viz. size, slope, roughness, alignment, etc., which determine the stage of a river at any given point of time for a given rate of flow. The length of the channel that is effective as a control, increases with the discharge and new features that affect the stage-discharge relation may enter at higher discharges. Generally, the flatter the stream gradient, the longer the reach of the channel control.

2.3 Partial Controls

A partial control is a control that acts in conjunction with another control in governing the stage- discharge relation. Such a situation exists where the section control is the sole control at lower stages and it is the channel control at the higher stages. At intermediate stages, there is a transition from one control to another and at these stages both section and channel controls act together as partial controls.

2.4 Complete Controls

A complete control is the one that governs the Stage- Discharge relation throughout the entire range of stages. Natural existence of such a control is of rare occurence and few artificial controls may act as complete controls. A section control could be a complete control if it is a weir, dam, cascade or falls etc., of such a height at which the downstream condtions do not affect the Stage-discharge relation. A channel control could be a complete control as in the case of a sand channel that is free of ruffles or bars, or an artificial channel like concrete lined flood way etc.

2.5 Compound Controls

A compound control is the one in which the stage- discharge relation is governed by more than one control, each of which may act at different ranges of stages. A common example is the usual situation where the section control is effective at lower stages, a partial control at intermediate stages and a channel control at higher stages.

2.6 Artificial Controls

These controls are manmade like a structure built across the stream, or constriction of the river width due to construction of a bridge or encroachment of the river banks by dumping waste material etc., which serve as a control for the stage-discharge relation. Artificial controls like weirs etc., eliminate many of the undesirable characteristics of natural controls. They are not only permanent but also provide stability to the stage-discharge relation.

Some of the important attributes that are desirable in an artificial control are:

☛ The control should be permanent and structurally stable.

☛ Excessive seepage under and around the structure should be avoided and if required, necessary precautions like sheet piling or construction of concrete cut-off walls should be undertaken.

☛ The crest of the structure should be as high as feasible so that the downstream conditions do not affect the control at higher stages.

☛ The profile of the crest of the control should be so designed that

a. Small changes in the discharges at low stages should be able to cause measurable change in the stage.
b. The stage-discharge relation obtained could be extrapolated to peak discharges with minimum error.

☛ The structure should be designed to be self- cleansing, so that the sediment being carried by the stream remains undisturbed.

2.7 Correlation and Regression

Both simple and multiple correlation and regression analysis are the oldest statistical techniques used in hydrology. The main objectives of this analysis are the transfer of information between points at which the same variable was observed or between two among several variables observed simultaneously. This includes the completion of missing data in Hydrology series and the prediction of a variable from the observed one or several other variables.

Correlation is defined as the association of two or more random variables that only partly explains the total variations of other random variables involved in the association equation. The effect of unaccounted or neglected random variables and errors is responsible for the remaining or unexplained part of the variation. Typical examples of correlative associations of random variables in hydrology are: Rainfall-runoff, Stage-Discharge, Sediment load- runoff, oxygen content- water temperature etc.,. An example of multiple correlation is to define run-off as a function of rainfall characteristics, river basin geometric properties, soil and vegetation factors, moisture conditions of the basin, and other factors.

Regression represents a mathematical equation expressing one random variable as being correlatively related to another random variable or to several random variables. All variables on the right side of this equation do not need to be random variables. The regression equation may be any function that can be fitted to set of points of observed variables. The selection of the function to be fitted to points determines the type and the degree of correlative association. Determining mathematical models of correlative association of two or more variables, so that the best prediction of one variable can be obtained from other variable or variables is referred to as regression analysis, and the models are called regression functions.

3.0 CHARACTERISTICS OF RATING CURVES

The stage-discharge relations are usually developed from a graphical analysis of the discharge measurements plotted on either rectangular coordinate or logarithmic sheet. The discharge values are plotted as the abscissa and the corresponding stage values as the ordinate and a curve (in case of rectangular coordinate plot) or a straight line (in case of logarithmic plot) is fitted.

Mathematically, the relation that controls the stage- discharge relation is of the form

$$\mathbf{Q = C(G-G_0)^n} \qquad \text{where}$$

Q is the discharge (Cumecs)

G is the gauge height (Metres)

G_0 is the gauge height for zero discharge (Metres)

C is the station constant

n is the slope of the rating curve

Such an equation plots as a straight line on a logarithmic plot of $\log(G-G_0)$ Vs $\log(Q)$. Plotting the stage-discharge curve on a log-log plot has its advantages. Since the shape of the cross section and accordingly the control varies with the stage, the stage-discharge points plot as straight lines with changing slopes for different ranges of stages thereby indicating accurately the stage at which the control changes. Also the portion of the rating curve that is applicable to any range of stage can be linearized for extrapolation or interpolation. A typical plot of stage-discharge curve on rectangular and logarithmic scale is shown at Figures 1 and 2 respectively.

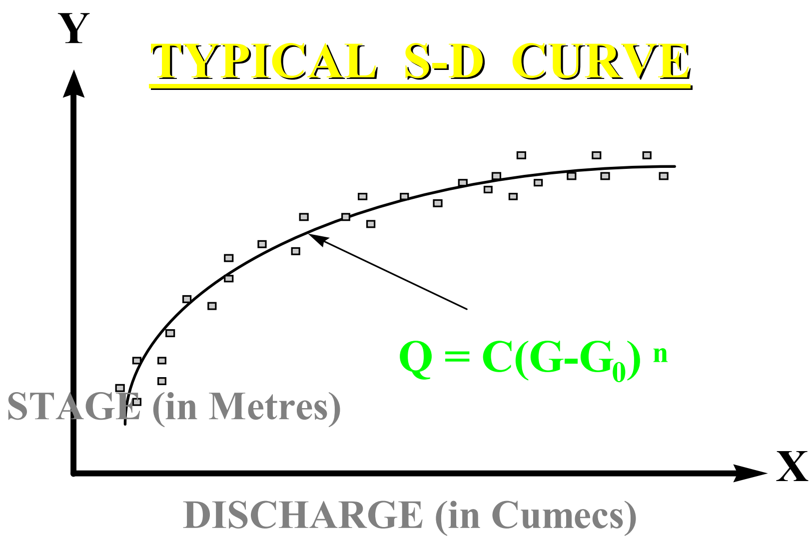


FIGURE 1

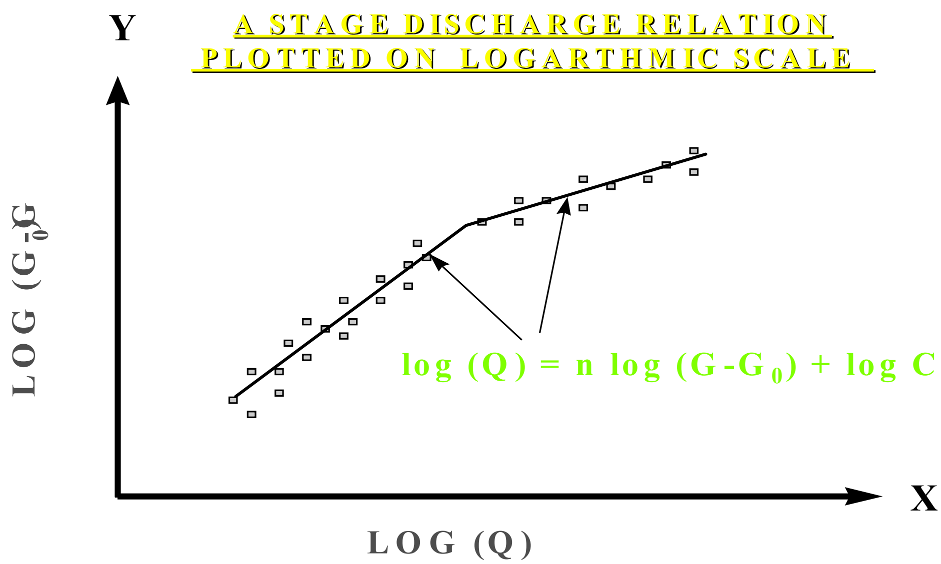


FIGURE 2

3.1 Determination of stage for Zero Discharge

3.1 1 Trial & Error Method

The most important aspect in plotting the log-log plot is the determination of the value of 'G_0' i.e. the gauge for zero discharge. The value of G_0 can be determined by trial and error. If the value of the G_0 selected is greater than the actual, the curve on log- log plot would be concave upwards and on the other hand, if it is less than the actual, the curve would be concave downwards and the actual value can then be determined by adjusting ' G_0' such that the points plot as a straight line.

3.1.2 Analytical Method

A more objective method to determine the value of G_0 is described in Section A-4.9 of IS:2914-1964 "IS Recommendations for Estimation of Discharges by Establishing Stage-Discharge Relations in Open Channels" which is discussed here.

Select three sets of values namely (G_1, Q_1), (G_2,.Q_2) and (G_3,Q_3) on the rating curve, on the ordinate scale such that the values Q_1, Q_2 and Q_3 are in geometric progression i.e.

$$Q_2^2 = Q_1 \times Q_3 \quad \text{---------------------} \quad (1)$$

The values of G_1 , G_2 and G_3 corresponding to Q_1 , Q_2 and Q_3 are also picked up. In accordance with the properties of a straight line of the form $Q = C\,(G- G_0)^n$ and substituting in equation (1) above, it can be derived that

$$(G_2- G_0)^2 = (G_3- G_0) \times (G_1- G_0) \quad \text{--------------------------} \quad (2)$$

Expanding equation (2) and solving for Go yields

$$G_0 = \frac{G_1G_3 - G_2^2}{G_1 + G_3 - 2G_2} \quad \text{--------------------------} \quad (3)$$

Thus the value of G_0 can be computed.

3.1.3 Graphical Method

A graphical solution of obtaining the value of G_0 is described now.

As above, three values of discharge in geometric progression are selected. Let the corresponding points be A,B and C as illustrated in figure 3. Through points A and B vertical lines are drawn and through points B and C horizontal lines are drawn to meet the verticals at points D and E respectively. Join ED and BA and extend so that they intersect at F. Then the ordinate of F is the value of G_0.

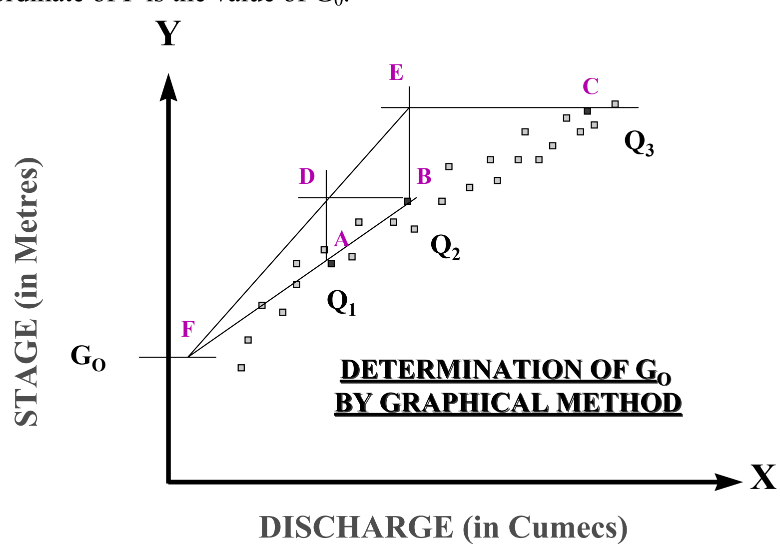


FIGURE 3

Thus the value of G_0 can be obtained by using any of the three methods described above.

4.0 REGRESSION ANALYSIS

After having selected the value of G_0, the constants C and n of the mathematical relation can be determined by statistical method, i.e. regression analysis.

For least square regression, the sum of the squares of the deviations between Log(Q) and Log $(G- G_0)$ should be minimum. Accordingly, it can be shown that

$\Sigma(Y) - N \text{ Log } (C) - n\, \Sigma (X) = O$ and

$\Sigma(XY) - \Sigma(X) \text{ Log } (C) - n\, \Sigma(X)^2 = O$ where

$\Sigma(Y)$ = Sum of all values of Log (Q)

$\Sigma(X)$ = Sum of all values of Log $(G- G_0)$

$\Sigma(X)^2$ = Sum of the squares of X

$\Sigma(XY)$ = Sum of the products of Log(Q) and Log$(G- G_0)$

N = Number of observations

The preparation of the data and solution of the equation is simplified by employing a tabulation procedure as detailed in the handout .

5.0 EXTRAPOLATION OF RATING CURVES

If the discharge measurements cover the entire range of stages experienced during a period, and the stage-discharge relation is stable, there is little or no problem in defining the discharge rating for that period. On the other hand, if, as is usually the case, there are no discharge measurements to define a part of the curve, then the defined part of the curve needs to be extrapolated to the highest or lowest stage experienced as the case may be to find the discharge at that stage. The stage-discharge relation curves are primarily intended for interpolation and their extrapolation beyond the highest recorded discharge or lowest recorded discharge may be subject to risk and indefinite errors. Physical factors like over-bank spills at higher stages, shifts in controls at very low and very high stages, changes in rugosity coefficients at different stages etc., materially affect the nature of stage-discharge relationship at the extreme ends and all these factors are to be taken into account while extrapolating the stage-discharge curve. Such extrapolations are always subject to error, but these errors can be minimized by proper application of hydraulic principles and hence it is always better to check the results obtained by more than one method. Extrapolation of rating curves can basically be classified as "Low flow extrapolation" and "High flow extrapolation".

5.1 Low flow extrapolation

Low flow extrapolation is best performed on a rectangular co-ordinate graph plot because the co-ordinates of zero flow can be plotted on such paper. It is to be noted that zero flow cannot be plotted on Logarithmic paper. An example of low flow extrapolation is demonstrated in Fig. 4 where the circled points represent discharge measurements plotted on the co-ordinate scale of stage vs discharge. The stage for zero flow can be obtained by field observations or by the method described in the preceding section. After identifying the stage for zero discharge, the point of zero flow is joined by a smooth curve to the defined part of the rating curve.

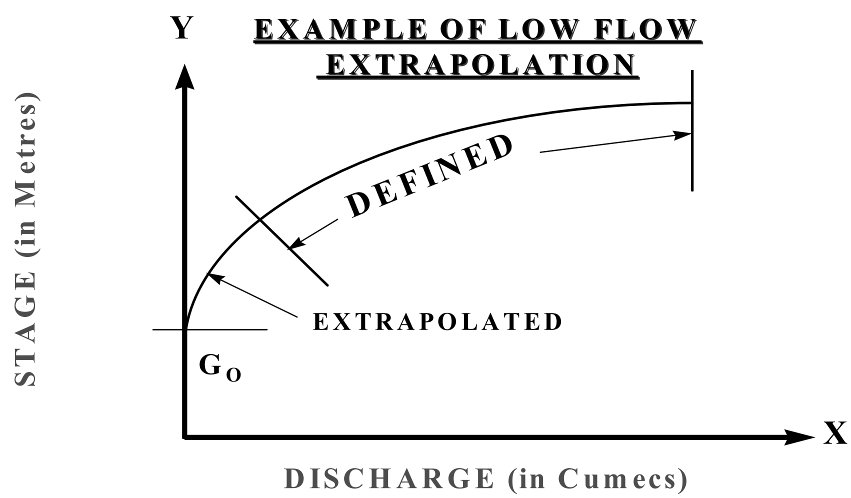


FIGURE 4

5.2 High flow extrapolation

High flow extrapolation is usually complex and great care is to be exercised in arriving at the extrapolated values. In the following sub-sections different methods of extrapolation are described and their practical utility is also discussed. It is suggested that the best method suiting the practical conditions should be selected for extrapolation

5.2.1 Double log extension

This method involves simple extension of the straight line plot on log-log sheet to the required gauge height and reading the corresponding discharge. This method is usually not reliable because of the likelihood of change in control at high stages. The change in control will result in a change in the slope of the straight line plot and the discharge read from the extended plot of log(G- G_0) Vs log(Q) may lead to serious errors in estimating the peak flows.

This method, due to its simplicity (if log-log plot is readily available) could be used for obtaining a rough and conservative value of the estimated discharge provided the control is not changing significantly.

5.2.2 Steven's Method

The Steven's method is based on the equation of steady flow i.e. Chezy's equation which is

$Q = A.C. \sqrt{R.S.}$ with usual notations

If $C\sqrt{S}$ is assumed to be constant for a station and D, the mean depth is substituted for R, then

$Q = K.A.\sqrt{D}$ where $K = C\sqrt{S}$

Known values of Q and $A\sqrt{D}$ are plotted on a rectangular coordinate graph and such plotting usually defines close to a straight line, which can be easily extended for extrapolation. Values of $A\sqrt{D}$ for the stages above the existing rating can be obtained from field measurement and used with the extended curve for estimating the discharge. It is important to note that an abrupt discontinuity in the curve is likely at bankful stages.

5.2.3 Conveyance slope method

This method is based on the equations of steady flow, such as Chezy's or Manning's equations. These equations can be expressed as

$Q = KS^{1/2}$ Where

Q is the discharge

S is the slope and

K is the conveyance

In Manning's equation,

$K = 1/n\ A.R.^{2/3}$ and

in Chezy's equation

$K = C.A.R.^{1/2}$ where

A is the area of the cross section

R is the hydraulic radius and

C & n are Chezy's and Manning's constants respectively

In the above equations, the values of 'A' and 'R' can be obtained from a field survey and the value of 'C' or 'n' can be estimated. Thus, the value of 'K' embodying all the elements which can either be measured or estimated can be computed for any given stage. The accuracy of 'K' depends on the errors involved in estimating the values of 'C' or 'n' which are usually not critical. The values of 'K' covering the complete range of stages upto the required peak level are computed and conveyance' curve is obtained by plotting 'K' as abscissa and stage values as ordinate on rectangular co-ordinates and joining the points by a smooth curve. A typical conveyance curve is shown in Fig 5.

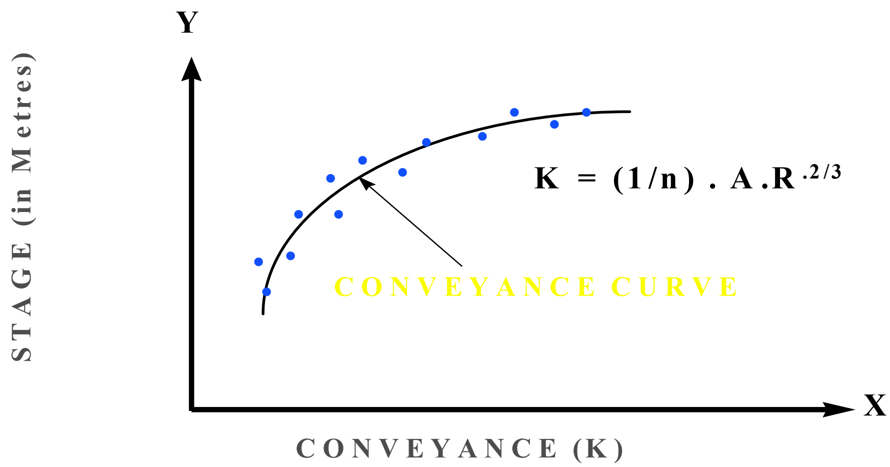


FIGURE 5

The 'slope curve' is obtained by plotting the values of slope 'S' as abscissa and the corresponding stage as ordinates on a rectangular co-ordinate graph paper and joining the points thus obtained by a smooth curve. The values of slope for the purpose can be obtained from the conveyance equation for measured values of discharge and from the extrapolated conveyance curve for the unmeasured range of stages upto the peak stage. The extrapolation is guided by the fact that the slope tends to become constant at higher stages and this constant slope is the normal slope or the slope of the stream bed. If the upper end of the defined part of the slope curve indicates that constant or near constant value of 'S' has been attained as shown in Fig. 6, the extrapolation can be made with confidence. However, if the upper end of the defined part of the slope curve has not reached a stage where 'S' has a near constant value, the extrapolation is subject to uncertainty and in that situation, the general slope of the stream bed as determined from the topographic map, should provide a guide to the probable constant value of slope to be attained at the higher stages.

The value of discharge at any particular stage can now be obtained by multiplying the value of 'K' from the conveyance curve and S computed from the corresponding values of 'S' from the slope curve and thus the upper portion of the stage-discharge relation can be constructed. It is highly unlikely that the value of 'S' extrapolated will be with an error of $\pm$ 10% and even if an error of $\pm$ 10% is committed in estimating 'S', the error in estimate of $\sqrt{S}$ and hence error in estimating 'Q' will be of the order of less than 5%. The likelihood of decrease in slope at high stages, as shown by the dotted curve to the left side of the slope curve, in Fig.6, is greatest when the over bank flow occurs.

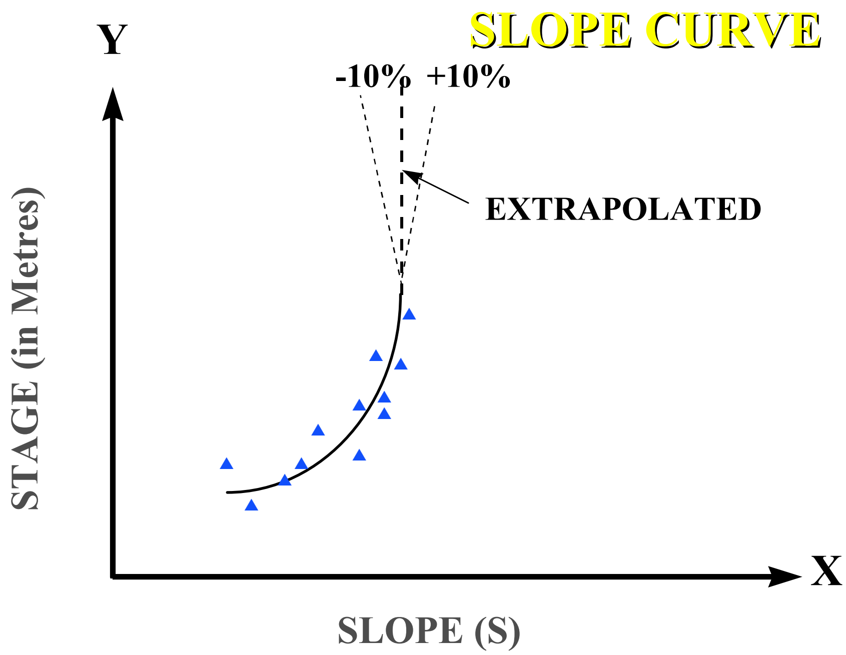


FIGURE 6

This method is recommended for use because of its simplicity and also as an accurate value of discharge with minimum error can be obtained. This method, because of its superiority has supplanted the earlier methods, one of them being the Steven's method i.e. Q vs A√d method.

5.2.4 Areal comparison of peak runoff rates

This method can be used to determine peak discharges at a gauging station from the known peak discharges at the surrounding stations, when flood stages are produced over a large area by an intense general storm. The known peak discharges are converted to peak discharge per unit of catchment area and expressed in terms of cubic metres per second per square kilometre. If there has been relatively little difference in storm intensity over the area affected, the peak discharge per unit area may be correlated with the catchment area of the gauging station under consideration. If the storm intensity is variable, the correlation will require the use of some index of storm intensity as third variable.

This method is not recommended for use to obtain the final and accurate value of peak discharge but could be used to obtain peak discharges which act as guide in extrapolating the rating curve at a gauging station.

5.2.5 Flood routing

Flood routing techniques may be used to test and improve the overall consistency of records of discharge during major floods in a river basin. The number of direct observations of discharge during such flood periods is generally limited by the short duration of the flood and the inaccessibility of certain stream sites. Through the use of flood routing techniques, all observations of discharge and other hydrological events in a river basin may be combined and used to evaluate the discharge hydrograph at a single site.

The resulting discharge hydrograph can be then used with the stage hydrograph for that gauge site to construct the stage-discharge relation for the site; or, if only peak stage is available at the site, the peak stage may be used with the peak discharge computed from the hydrograph to provide the end point for the rating curve extrapolation.

5.2.6 Step backwater method

The step backwater method is a technique in which the water surface profiles for selected discharges are computed by successive approximations. The computations start at a cross-section where the stage-discharge relation is defined or assumed, and proceed to the study site, which is the hydrological station whose rating requires extrapolation. If the flow is in the sub critical regime, as is usually the case in natural streams, the computations must proceed in the upstream direction; and vice versa when the flow is super critical.

Under conditions of sub critical flow, the computations start at the rating defined section and proceed upstream, subreach by subreach (in "steps"). If an initial cross section for the computation of water surface profile is selected far enough downstream from the study site, the computed water surface level at the study site, corresponding to any given discharge, will have a single value regardless of the stage selected for the initial site (Usually applicable when the initial site selected happens to be a dam or a gated structure). After the initial site is selected, the next step is to divide the study reach i.e., the reach between the initial site and study site, into subreaches. This is done by selecting cross sections where major breaks in the high water profile are expected to occur because of the changes in channel geometry of roughness. These cross sections are the end sections of the subreaches. The cross sections are surveyed and roughness coefficients are selected for each subreach.

The first step in the computations is to select a discharge Q at the initial section and obtain the corresponding stage with that value of discharge. Step back water computations are then applied to the subreach, which are based on steady flow equations viz Chezy's or Manning's equations, after the equations are modified for non uniformity in the subreach by use of the difference in velocity head at the end cross sections. The Chezy's equation is related to the Manning's equation by the formula.

$$C = 1/n \ \ R^{1/6}$$

where n is Manning's roughness coefficient and R is hydraulic radius. The difference in water surface elevation (h) between the upstream section (subscript 1) and downstream section (Subscript 2) is given by

$$h = h_1 - h_2 = \frac{(L)\, V_1\, V_2}{C^2 R_1^{1/2} R_2^{1/2}} + \frac{(a_2 V_2^2 - a_1 V_1^2)^{(1+k)}}{2g}$$

where

h_1 and h_2 are the stages(at section 1 and at section 2)
L is the length of subreach
V_1 and V_2 are the average velocities in the crosssection (at section 1 and at sub section 2.)

g is the acceleration due to gravity
k is a constant
a_1 and a_2 are the velocity head coefficients whose value can be usually taken as 1.10 for rivers in India.

A trial value of stage for discharge Q is selected for the upstream section and the values of A,V and R are computed for both the sections. These values are then substituted in the above equation and after solving for 'h', the computed value of h is compared with the difference between the trial value of the stage at upstream section and the known stage at the downstream section. Seldom will the two values agree after a single computation and the procedure is repeated till the values agree. After the value of the stage is computed for the upstream section, that cross section becomes the downstream section for the next subreach upstream. Computations similar to those described above are repeated for that subreach, and for each subreach, till the cross section under consideration (of the study site) is reached, to provide a water surface profile extending to the study site.

The step backwater method can be used to prepare a preliminary rating for a new gauging station. This method can be put to best use, if computations are carried out by a computer. Different values of stages for the downstream section are selected so that a smooth curve can be fitted to the logarithmic plot of the discharge values at the new site. This preliminary rating can be revised, as necessary, when subsequent discharge measurements indicate the need for such a revision.

6.0 SHIFTS IN DISCHARGE RATINGS

Stage-discharge relations are usually subject to random fluctuations which result in shifts of the discharge rating. These shifts indicate that the stage- discharge relations are not permanent but vary from time to time, either gradually or abruptly, because of the changes in physical features that form the control for the gauging station. The fluctuations in the stage- discharge relations result from the dynamic force of moving water, and as it is virtually impossible to sort out the minor fluctuations, a rating curve that averages the measured discharges within close limits is considered adequate. It is also imperative that the discharge measurements are not error free, and consequently an average curve drawn to fit a group of measurements is probably more accurate than any single measurement.

If a group of consecutive measurements subsequently plot to the right or left of the average rating curve, it is clearly evident that a shift in the rating curve has occurred. If, however, only one or two measurements depart significantly from the average curve, then these measurements could be due to a random error.

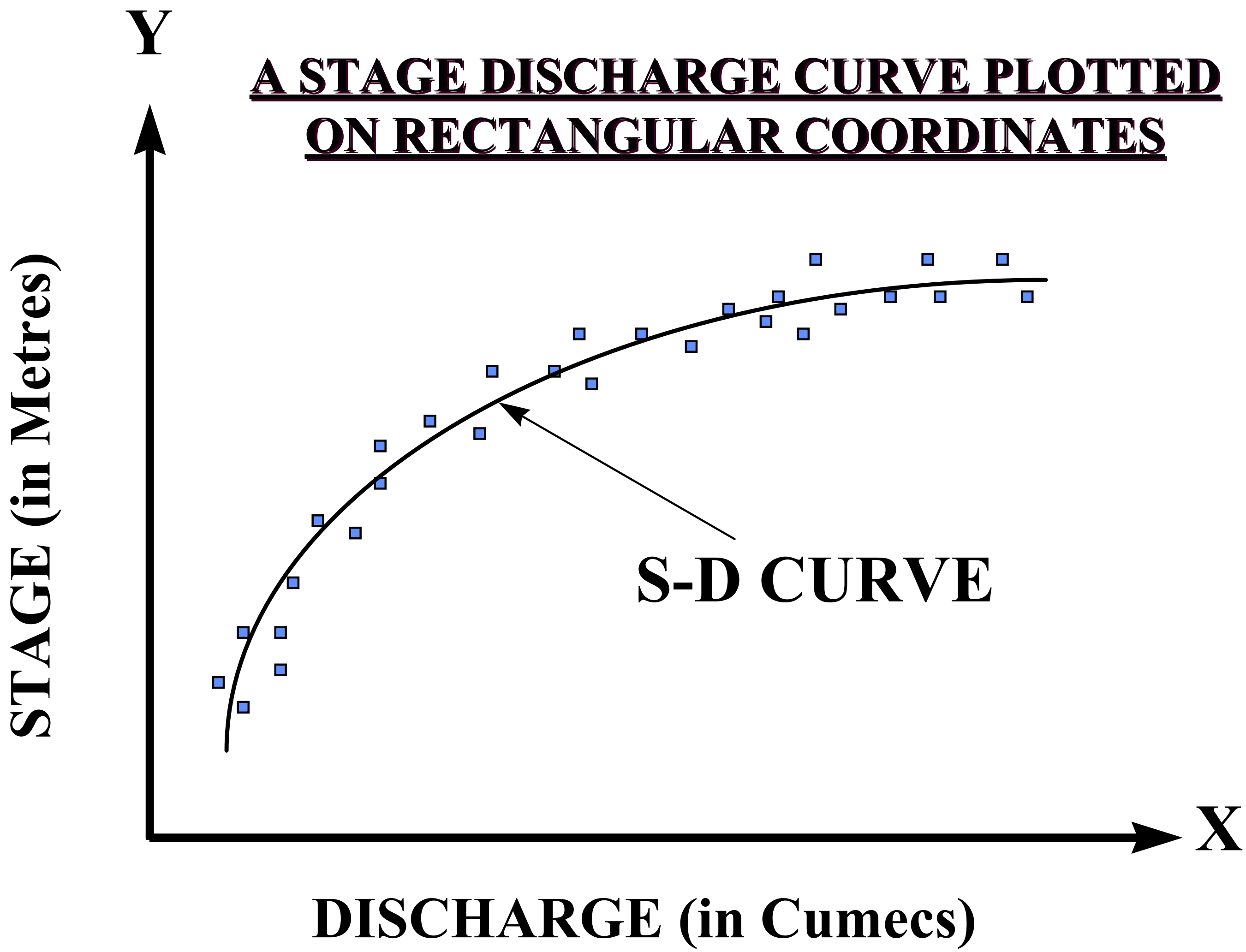
6.1 Rating shifts for natural section controls

The primary cause for changes in natural section controls is the high velocity associated with the high discharge. While the sections with a rock ledge outcrop will be unaffected by the high velocities, the sections with boulder, gravel and sandbar ruffles are likely to be affected. After a flood, the ruffles are often altered so drastically as to bear no resemblance to their pre flood state, requiring a new stage-discharge relation to be defined. The shift curve ideally should be defined by currentmeter discharge observations. If the shift rating is plotted on a rectangular coordinate graph it will tend to be a parallel to the original curve, either to the left or right. The extreme low water end can be extrapolated to the actual point of zero flow, as determined in the field when low water measurements are made. The shift rating on a logarithmic plot will be a curve that is either concave upward or downward, depending on whether the shift is to the left or right.

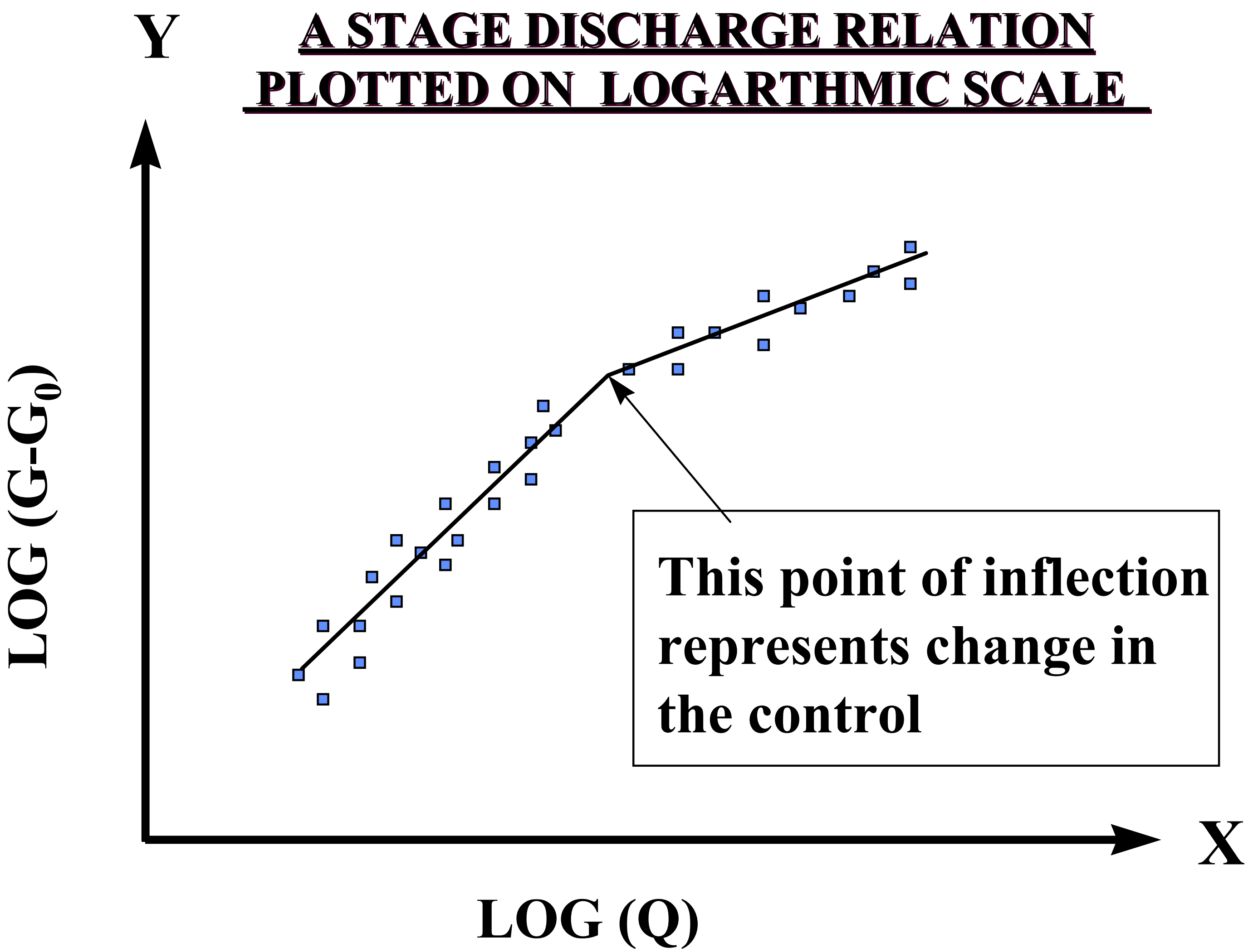
6.2 Rating Shifts for Channel Control

In natural streams, the shifts in the section control (for low stages) are usually accompanied by shifts in the channel control (for high stages). The most common cause of a shift in the channel control in a relatively stable channel is scour or fill of the stream bed caused by the high velocity flow. The scour usually occurs during a rise in the stream and the fill on recession. This is the result of the sediment transport process, which is a very complex process. The degree of scour in a reach is dependent not only on the magnitude of the discharge and velocity, but also on the sediment load coming into the reach. When scour is occurring in a pool at a meander bend, there is simultaneous filling at the crossover, or point of inflection between successive meander bends.

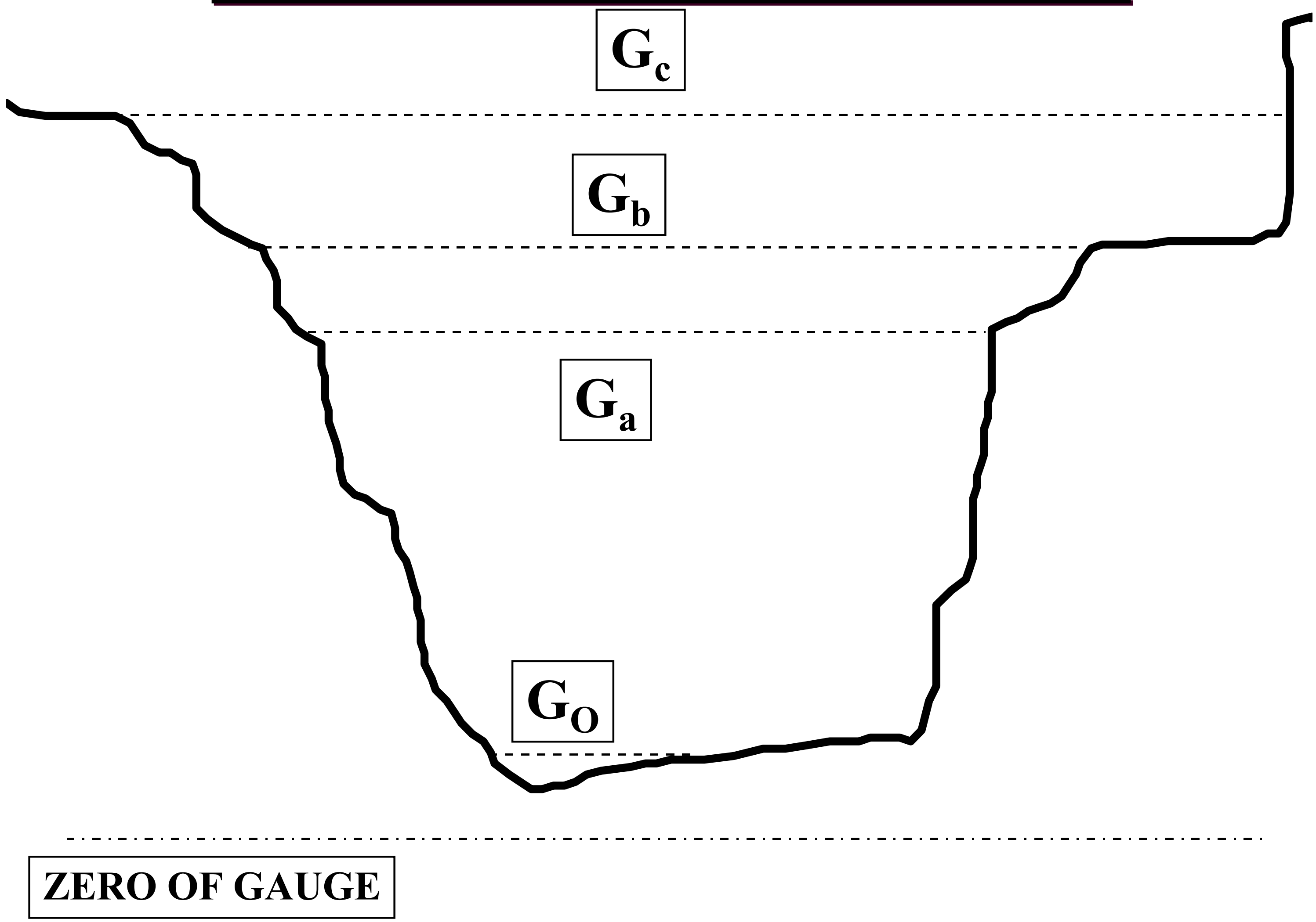
OVERHEAD SHEETS


Y
A STAGE DISCHARGE CURVE PLOTTED ON RECTANGULAR COORDINATES
STAGE (in Metres)
S-D CURVE
X
DISCHARGE (in Cumecs)

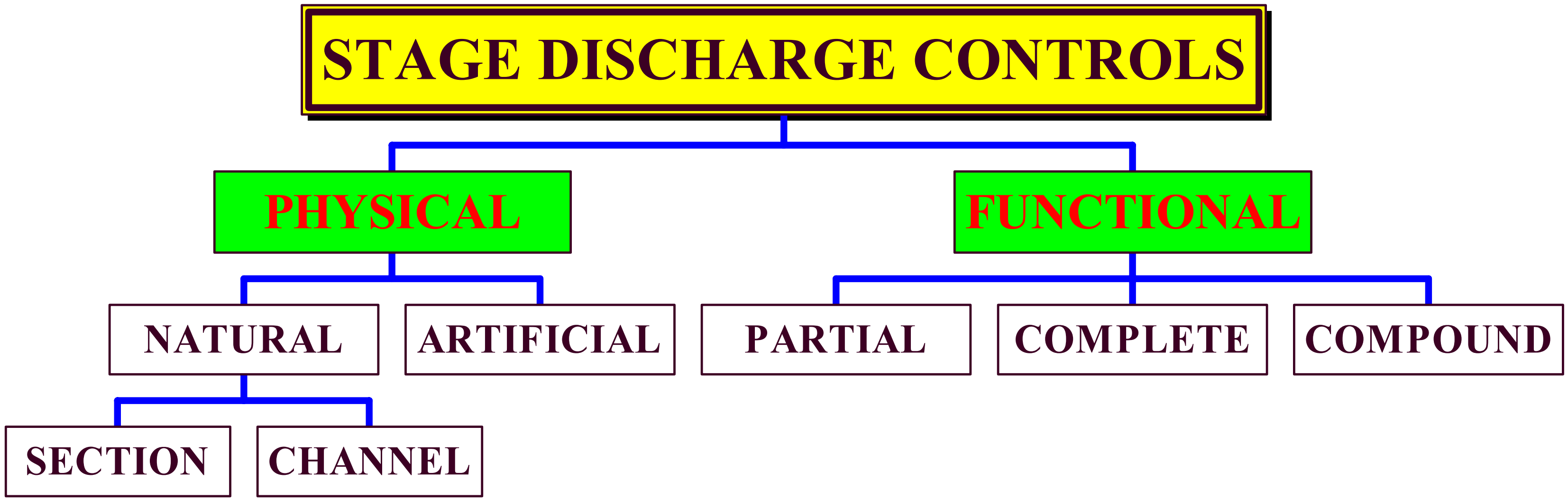
A STAGE DISCHARGE RELATION PLOTTED ON LOGARTHMIC SCALE



TYPICAL RIVER CROSS SECTION

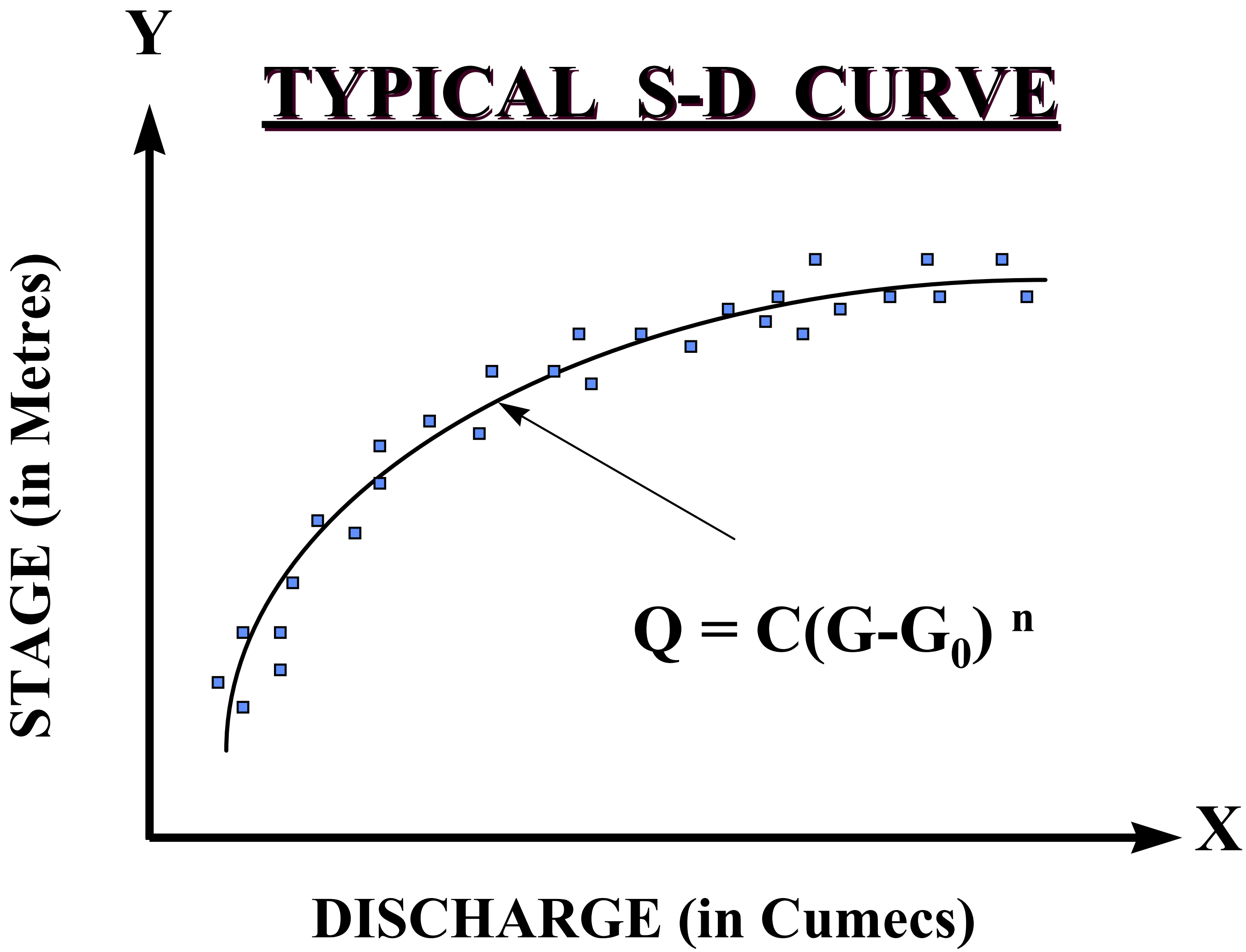


CLASSIFICATION OF STAGE DISCHARGE CONTROLS



IMPORTANT ATTRIBUTES OF ARTIFICIAL CONTROLS

- ✔ PERMANENT AND STABLE
- ✔ EXCESS SEEPAGE SHOULD BE AVOIDED
- ✔ D/S CONDITIONS SHOULD NOT EFFECT CONTROL AT HIGHER STAGES


TYPICAL S-D CURVE
Y
STAGE (in Metres)
$Q = C(G-G_0)^n$
X
DISCHARGE (in Cumecs)

TYPICAL STAGE DISCHARGE RELATION

$$Q = C(G-G_0)^n$$

where

Q is discharge in Cumecs

G is Gauge height in Metres

G_0 is gauge height for zero discharge

C is station constant and

n is the slope of the rating curve

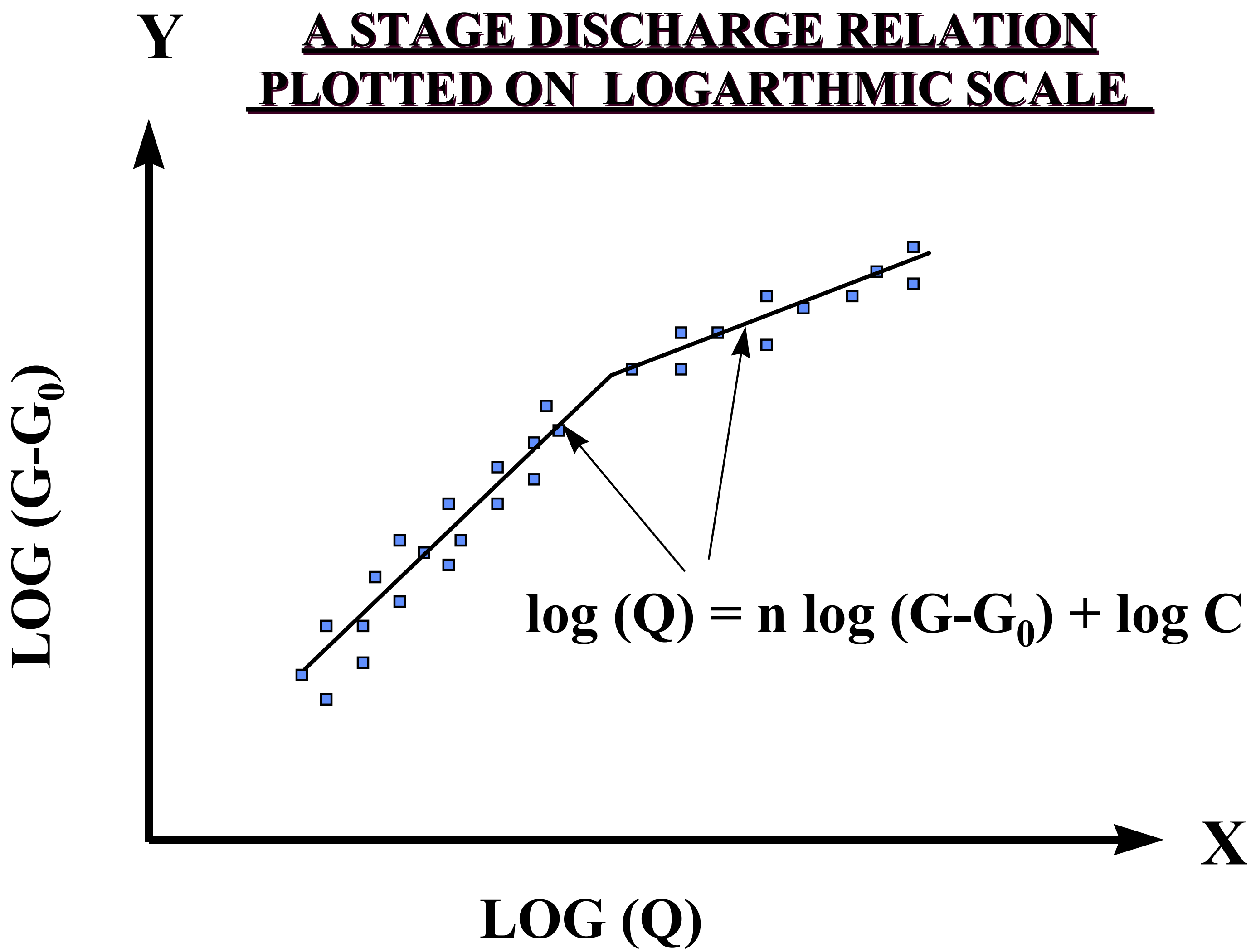
$$Q = C(G-G_0)^{n}$$

TAKING LOGARITHM ON BOTH SIDES WILL YIELD

$$\log (Q) = n \log (G-G_0) + \log C$$

which is a linear equation

A STAGE DISCHARGE RELATION PLOTTED ON LOGARTHMIC SCALE



IN THE EQUATION SHOWN IN THE PREVIOUS SLIDE THE UNKOWNS ARE

G_0 , 'n' & LOG (C)

TO FIND THESE, STATISTICAL METHODS ARE TO BE USED.

BUT BEFORE WE GO AHEAD, SOME DEFINITIONS

DEFINITIONS

CORRELATION	ASSOCIATION OF RANDOM VARIABLES
REGRESSION	REPRESENTS A MATHEMATICAL EXPRESSION RELATING TWO OR MORE RANDOM VARIABLES

HOW TO FIND G_0

- ☛ TRIAL AND ERROR METHOD
- ☛ ANALYTICAL METHOD

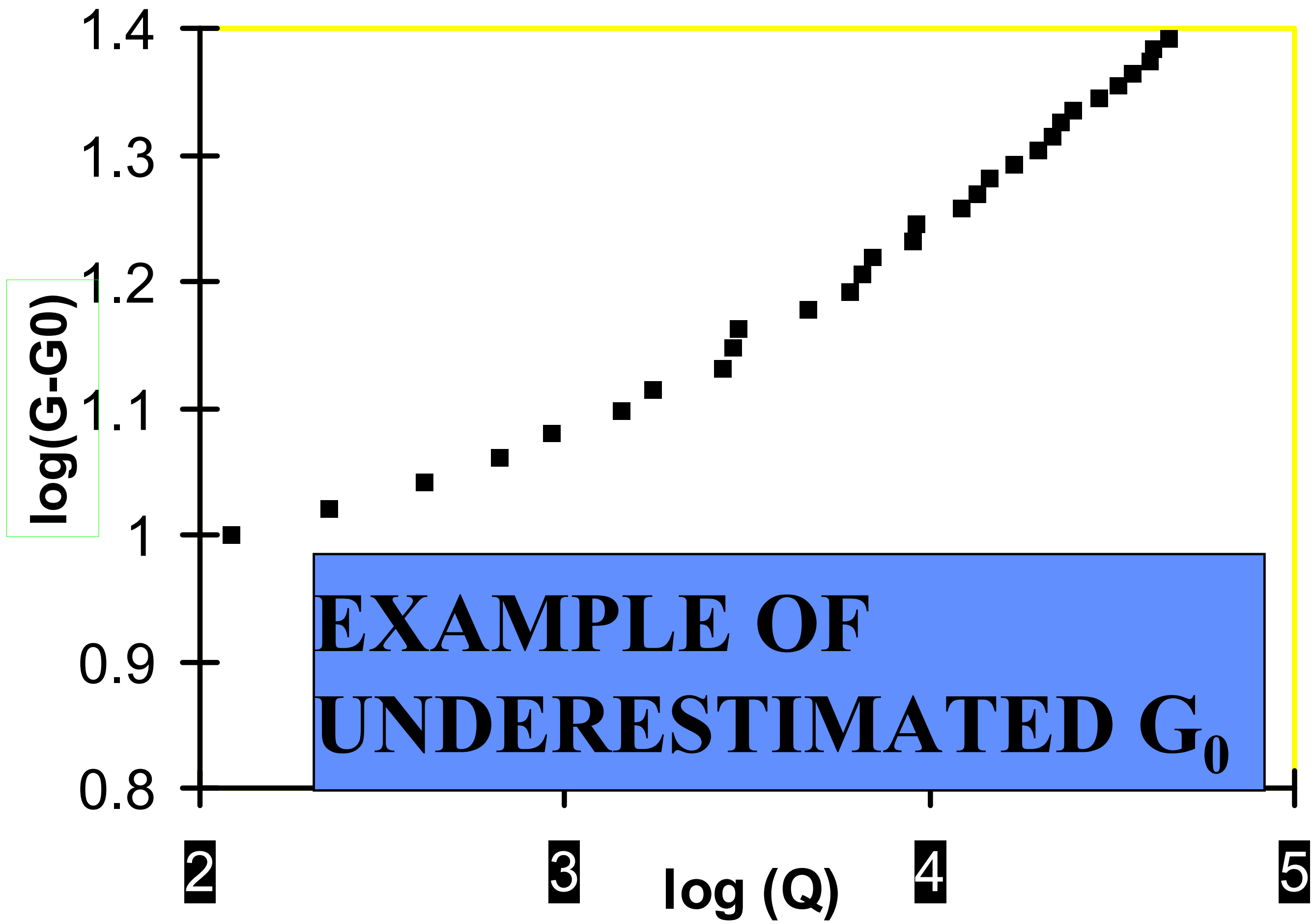
$$G_0 = \frac{G_1 . G_3 - G_2^2}{G_1 + G_3 - 2G_2}$$

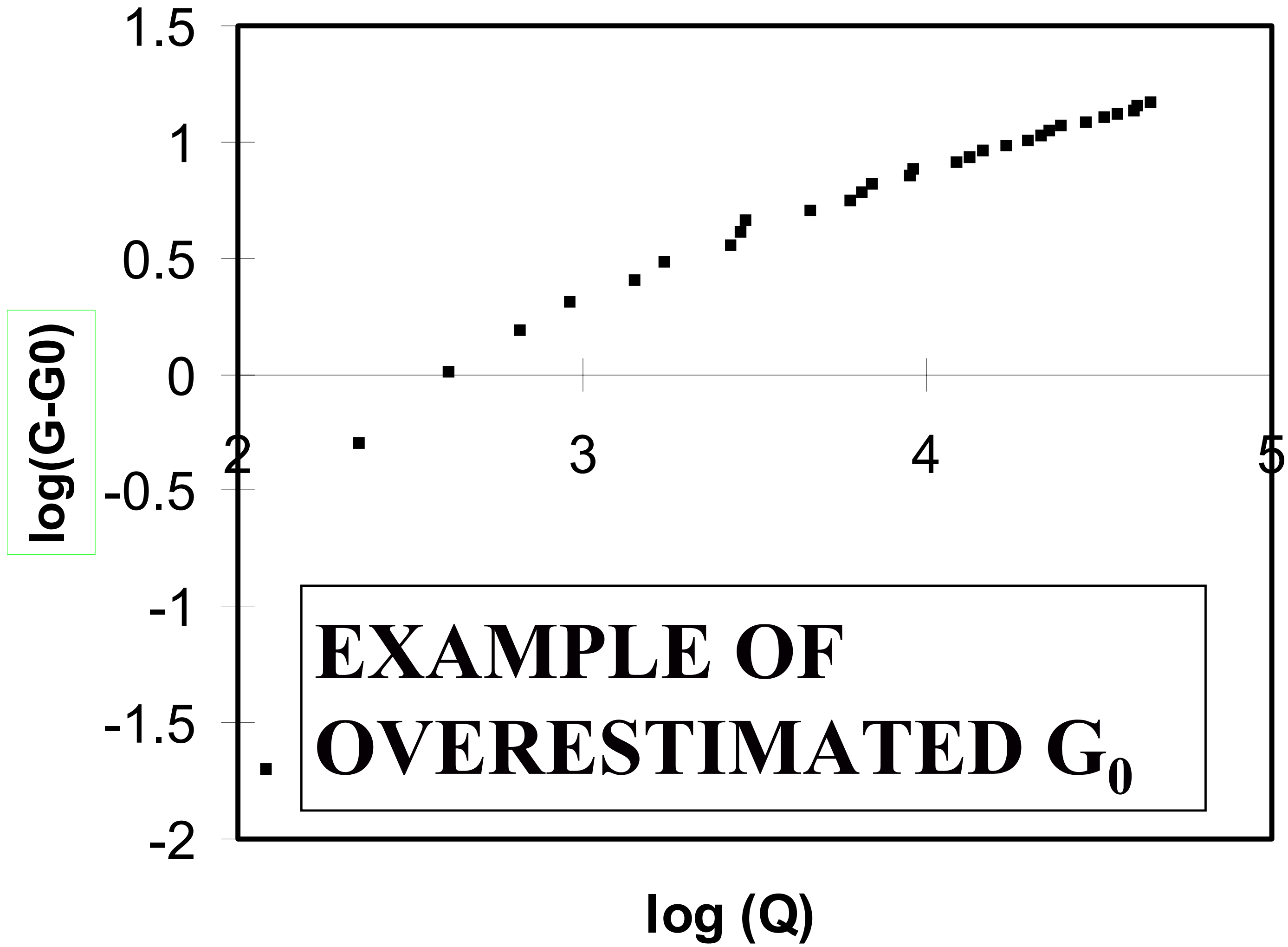
- ☛ GRAPHICAL METHOD

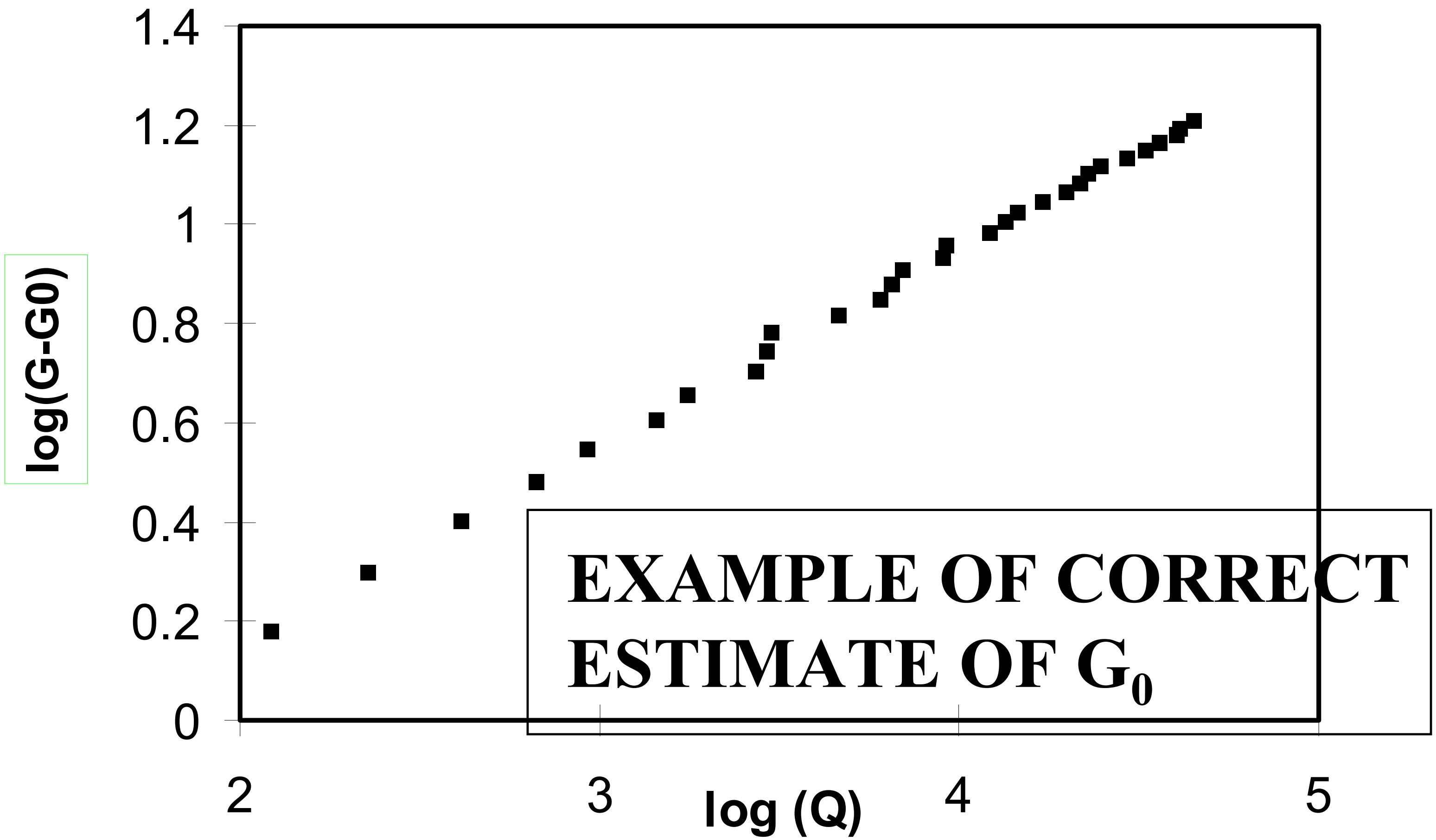
TRIAL AND ERROR METHOD

IN THIS METHOD, THE VALUES OF ‘Q’ & ‘G-G_0’ ARE PLOTTED ON LOG-LOG SHEET BY ASSUMING CERTAIN VALUE FOR G_0.

THE VALUE OF G_0 IS ADJUSTED TILL ALL THE POINTS TEND TO FALL ALONG A STRAIGHT LINE


1.4
1.3
1.2
1.1
1
0.9
0.8
log(G-G0)
2
3
4
5
log (Q)
EXAMPLE OF
UNDERESTIMATED G_0


1.5
1
0.5
0
-0.5
-1
-1.5
-2
2
3
4
5
log(G-G0)
log (Q)
EXAMPLE OF
OVERESTIMATED G0


1.4
1.2
1
0.8
0.6
0.4
0.2
0
log(G-G0)
2
3
4
5
log (Q)
EXAMPLE OF CORRECT ESTIMATE OF G0

ANALYTICAL METHOD

$$Q = C(G-G_0)^{n}$$

$$Q_1 = C(G_1-G_0)^{n}$$

$$Q_2 = C(G_2-G_0)^{n}$$

$$Q_3 = C(G_3-G_0)^{n}$$

SELECT THREE VALUES OF Q_1, Q_2 & Q_3 SUCH THAT THEY ARE IN GEOMETRIC SERIES,

i.e.,

$$Q_2^2 = Q_1 X Q_3$$

i.e.,

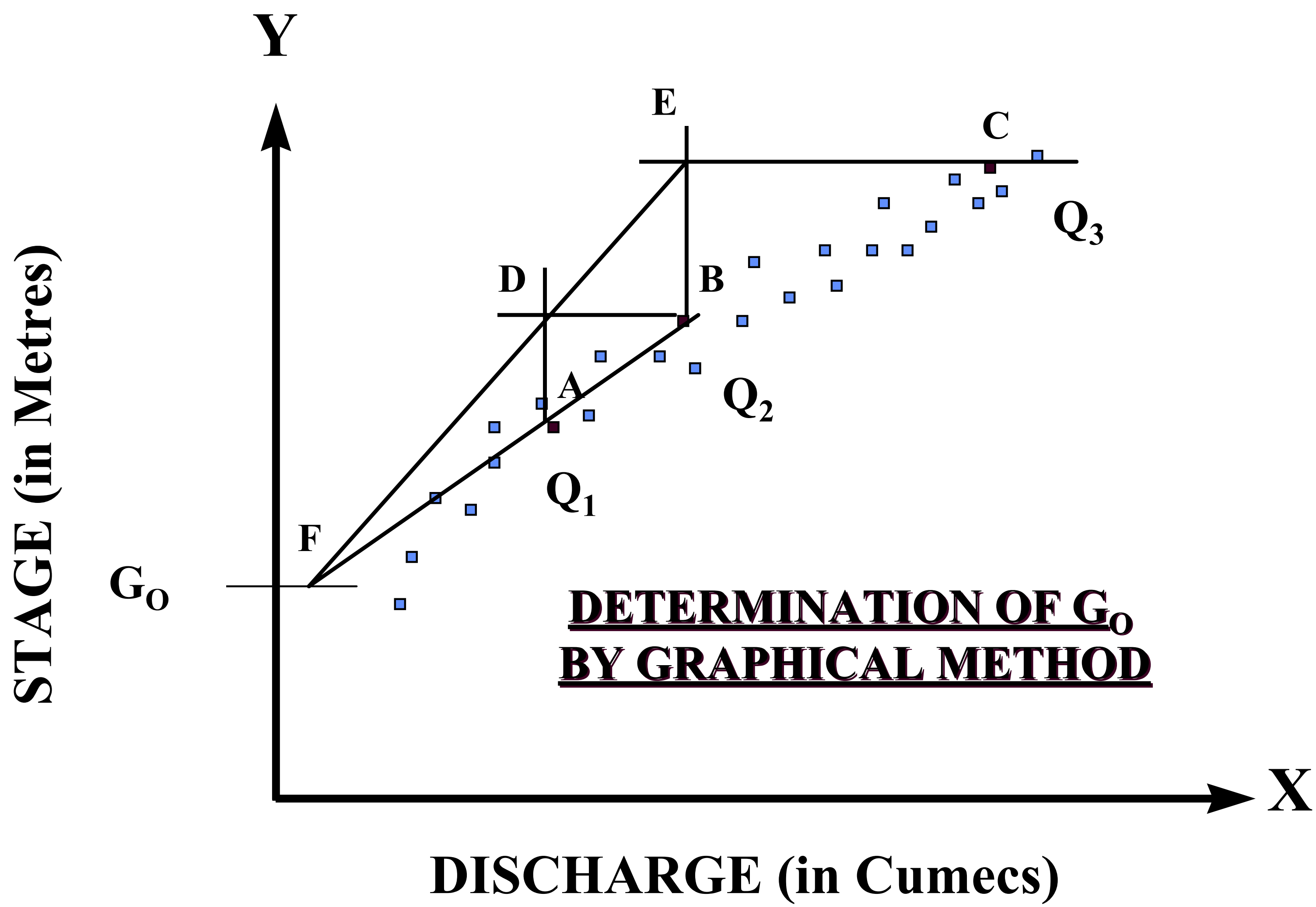
$$\{C\,(G_2-G_0)^n\}^2 = C(G_1-G_0)^n \times C(G_3-G_0)^n$$

$$C^2\,(G_2-G_0)^{2n} = C^2\,\{(G_1-G_0)\,(G_3-G_0)\}^n$$

$$\cancel{C^2}\,(G_2-G_0)^{2\cancel{n}} = \cancel{C^2}\,\{(G_1-G_0)\,(G_3-G_0)\}^{\cancel{n}}$$

$$(G_2-G_0)^2 = (G_1-G_0)\,(G_3-G_0)$$

SOLVING THE ABOVE EQUATION YIELDS THE VALUE OF G_0


Y
E
C
D
B
A
Q_3
Q_2
Q_1
F
G_O
STAGE (in Metres)
DETERMINATION OF G_O
BY GRAPHICAL METHOD
X
DISCHARGE (in Cumecs)

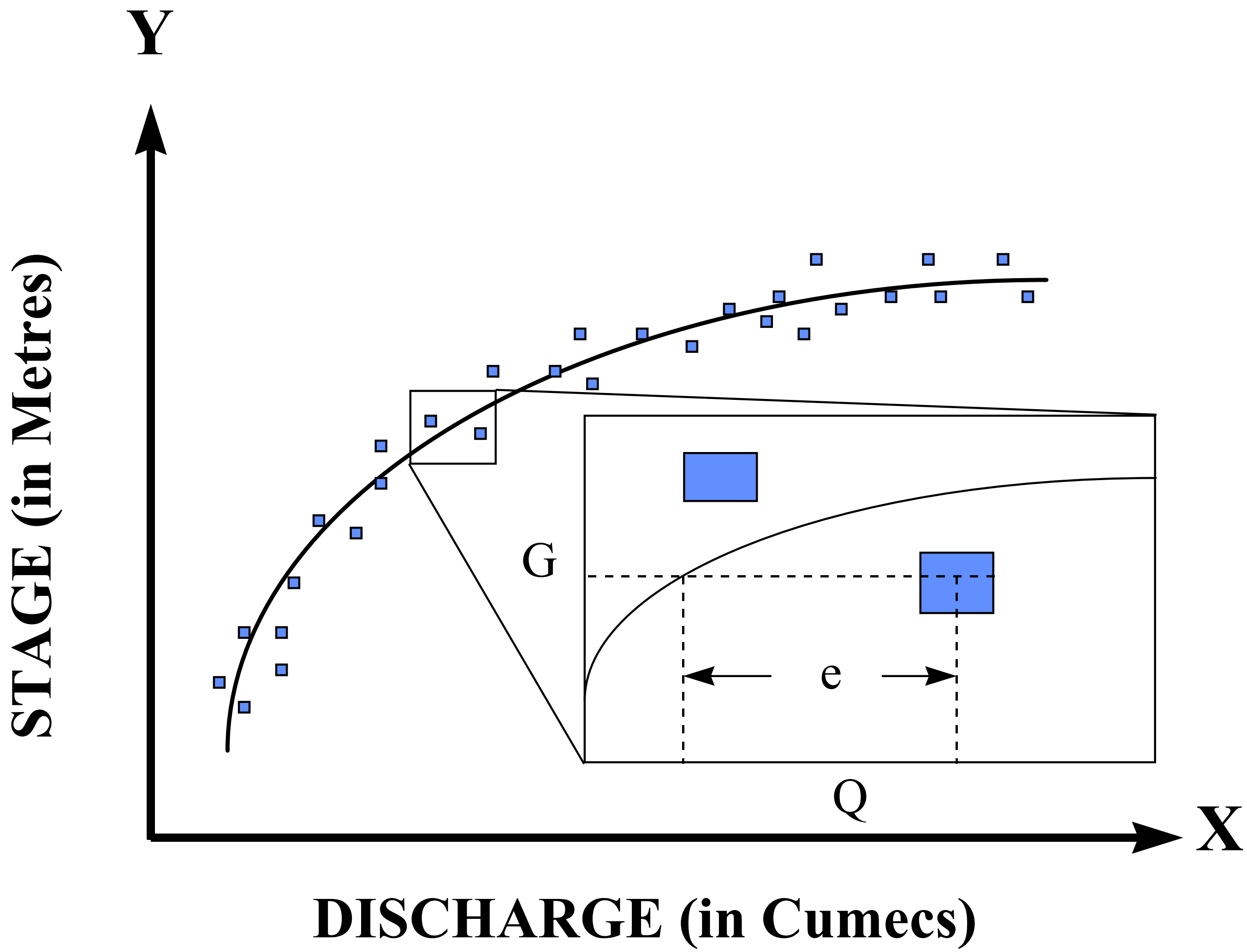
HOW TO FIND 'C' AND 'n'

The equation $\log (Q) = n \log (G-G_0) + \log C$
is of the form $Y = a + b X$

which can be solved using 'least squares method' based on Legendre's Principle.

LEGENDRE'S PRINCIPLE

" Sum of the squares of the residuals should be a minimum"


Y
STAGE (in Metres)
G
e
Q
X
DISCHARGE (in Cumecs)

e = observed value - computed value i.e.,

$$e_i = y_i - (a + b\,x_i)$$

$$E = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \{y_i - (a + b\,x_i)\}^2$$

since E is a minimum, partial differentiation w.r.t ‘a’ and ‘b’ will be equal to zero

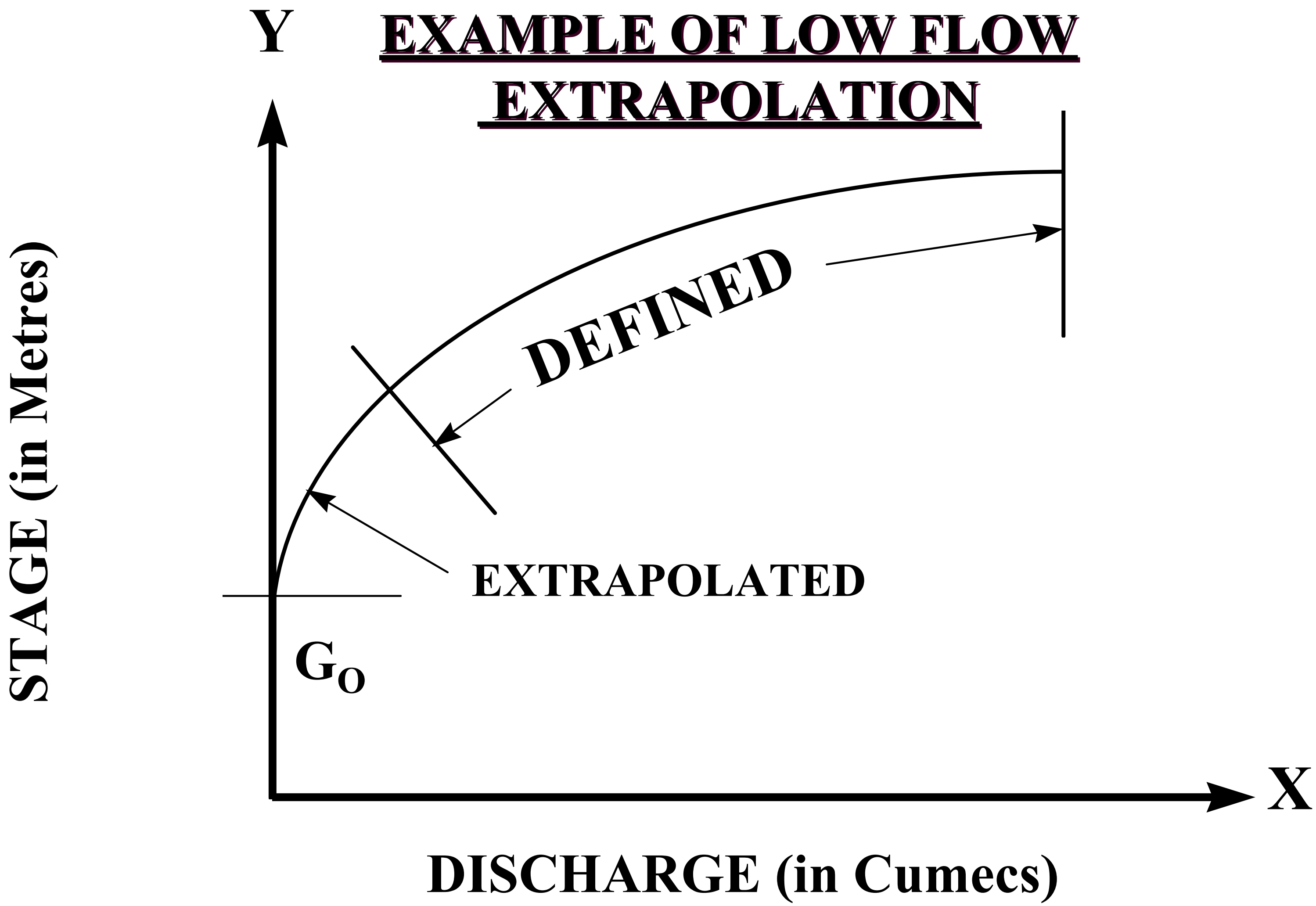
i.e..,

$$\frac{\delta E}{\delta \mathbf{a}} = 0 \quad \text{———————————} \quad (1)$$

$$\frac{\delta E}{\delta \mathbf{b}} = 0 \quad \text{———————————} \quad (2)$$

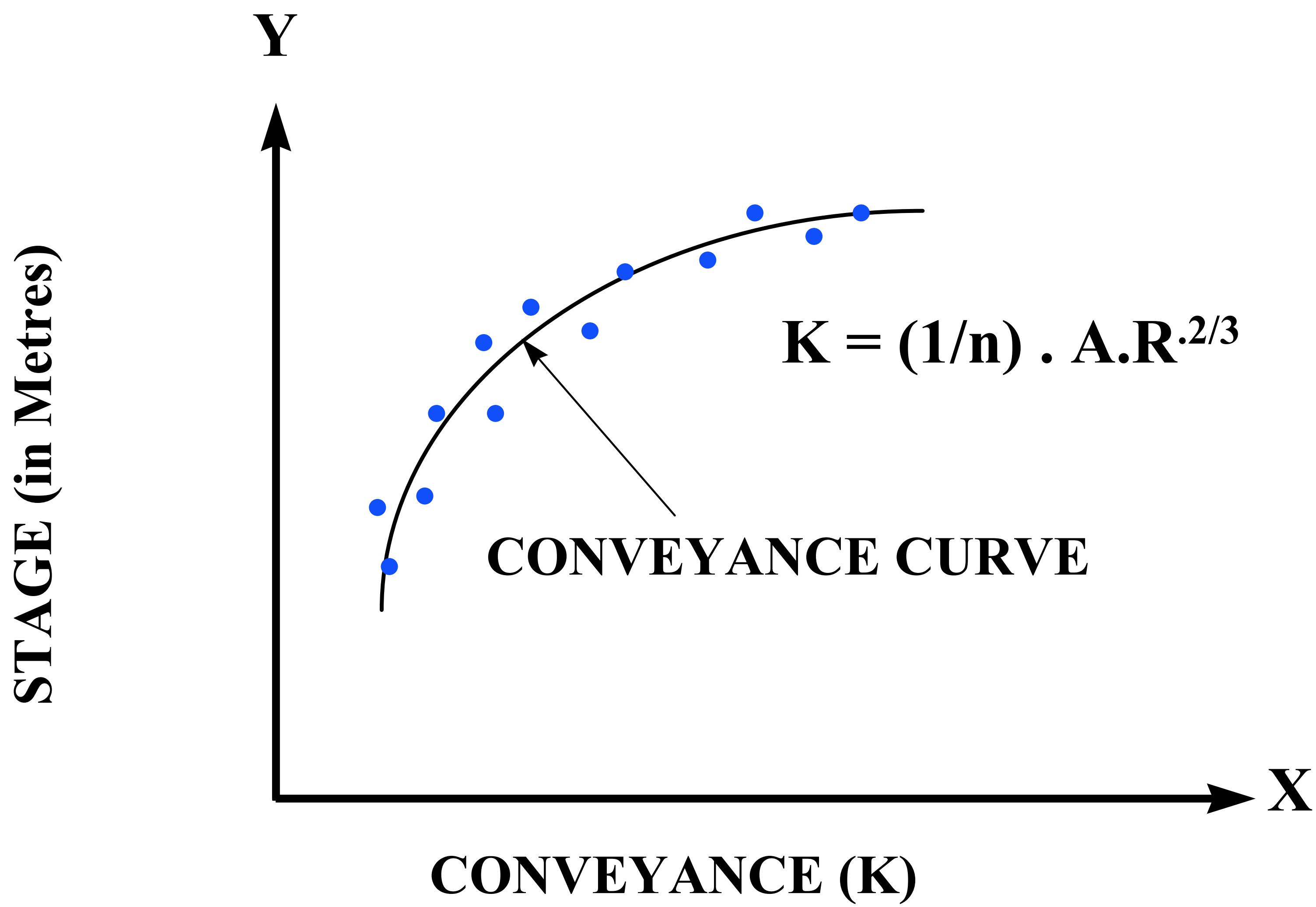
Solving equations (1) & (2) will yield 'a' & 'b'

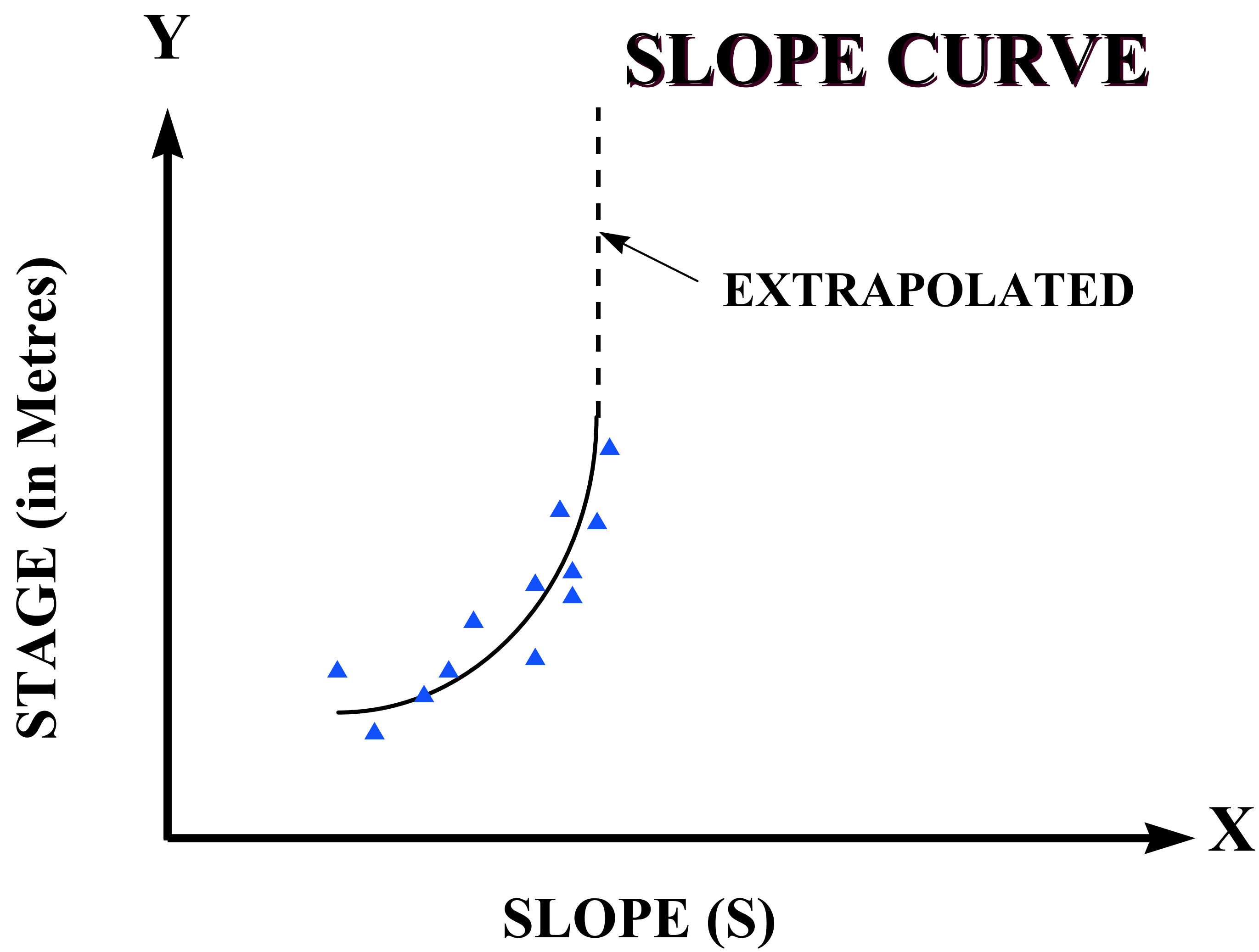
EXAMPLE OF LOW FLOW EXTRAPOLATION



HIGH FLOW EXTRAPOLATION

- ?? DOUBLE LOG EXTENSION ??
- STEVEN'S METHOD
- CONVEYANCE SLOPE METHOD
- AREAL COMPARISON OF PEAK RUNOFF RATES
- FLOOD ROUTING
- STEP BACKWATER METHOD


Y
STAGE (in Metres)
K = (1/n) . A.R.2/3
CONVEYANCE CURVE
X
CONVEYANCE (K)


SLOPE CURVE
Y
EXTRAPOLATED
STAGE (in Metres)
X
SLOPE (S)

FROM THE CONVEYANCE CURVE, THE VALUE OF ‘K’ CAN BE READ FOR ANY STAGE.

SIMILARLY, FROM THE SLOPE CURVE, THE VALUE OF SLOPE CAN BE READ FOR ANY GIVEN STAGE.

THE VALUE OF ‘Q’ IS GIVEN BY THE FORMULA

$$Q = K \sqrt{S}$$